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## CURRENT AND ELECTRIC-FIELD PRODUCTION NEAR A NONCONDUCTING

ROTATING SPHERE IN A HOMOGENEOUS PLASMA IN A STRONG MAGNETIC FIELD

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Central problems in magnetosphere physics include the motion of the plasma near the rotating earth and the currents and electromagnetic fields generated by this rotation. We consider the following model problem in order to obtain a conception of the structure of the current system and the motion of the plasma. An insulating sphere of radius $r_{0}$ is surrounded by a homogeneous incompressible conducting liquid and rotates with an angular velocity w. At the center of the sphere there is a magnetic dipole, whose moment coincides in direction with the axis of rotation. The plasma density $\rho$, conductivity $\sigma$, and viscosity $\mu$ are independent of the coordinates.

We assume that all the perturbations associated with the rotation decay away from the surface of the sphere. The attachment condition is obeyed at the surface of the sphere itself, while the normal component of the current becomes zero.

The behavior of the plasma is described by the equations of magnetohydrodynamics [1]

$$
\begin{equation*}
\rho\left(\mathbf{u}_{\nabla}\right) \mathbf{u}+\nabla\left(p+H^{2} / 8 \pi\right)=\left(\mathbf{H}_{\nabla}\right) \mathbf{H} / 4 \pi+\mu \Delta \mathbf{u} \tag{1}
\end{equation*}
$$

$$
\operatorname{div} \mathbf{u}=0, \operatorname{div} \mathbf{H}=0, \operatorname{rot}[\mathbf{u} \mathbf{H}]+v_{m} \Delta \mathbf{u}=0
$$

where $u$ and $H$ are the speed of the plasma and the magnetic field, $p$ is the plasma pressure, and $\nu_{m}$ is the magnetic viscosity, which is related to the conductivity $\sigma$ by

$$
v_{m}=c^{2} / 4 \pi \sigma
$$

The magnetic field within the sphere satisfies the equations

$$
\operatorname{rot} H^{i n}=0, \operatorname{div} \mathbf{H}^{i n}=0
$$

If the total magnetic field $H$ is represented as $H^{i n}=H_{D}+h$, where $H_{L}$ is the dipole magnetic field and $h$ is the perturbation field, then we introduce the scaler potential $\Phi$ for the magnetic field $h$ by $h=\nabla \Phi$, which gives

$$
\begin{equation*}
\Delta \Phi=0 \tag{2}
\end{equation*}
$$

The magnetic field does not change on passing through the surface of the sphere, so $\Phi$ should satisfy the following at the boundary:

[^0]$$
\partial \Phi /\left.\partial n\right|_{\tau=r_{0}}=\mathbf{H}_{n}-\mathbf{H}_{D n}, \quad \partial \Phi /\left.\partial \tau\right|_{r=r_{0}}=\left(\mathbf{H}_{\tau}-\mathbf{H}_{D \tau}\right) \cdot \boldsymbol{\tau},
$$
where $H_{n}$ and $H_{-}$are the normal and tangential components of the magnetic field outside the sphere and $n$ and $\tau$ are unit vectors normal and tangential to the sphere.

The first boundary condition with (2) having a known right part leads to a classical formulation, namely a Neumann problem for the Laplace equation, which has a solution unique apart from an additive constant. This means that the second boundary condition can in that case be used as an equation for $H_{\tau}: \mathbf{H}_{\tau}=H_{\tau}\left(H_{n}\right)$. In a spherical coordinate system with the $z$ axis lying along the axis of rotation, the condition for no current within the sphere can be put as $\partial H_{\varphi} \sin \theta /\left.\partial \theta\right|_{r=r_{0}}=0$ or $\left.H_{\varphi} \sin \theta\right|_{r=r_{0}}=f\left(r_{0}\right)=$ const. The requirement that $H_{\varphi}(\theta)$ is bounded for all values of $\theta$ means that $\left.H_{\varphi p}(\theta)\right|_{r=r_{0}}=0$.

The dipole field does not have a $\varphi$ component, so $\left.h_{\varphi}\right|_{r=r_{0}}=0$ which means that the total magnetic field and also the perturbed one lie in the meridional planes within the sphere.

The boundary conditions for the speeds of the plasma are obvious: The motion of the plasma should vanish far from the surface of the sphere, while the attachment condition applies at the surface itself. If $u, v$, and $w$ are the $r, \theta$, and $\varphi$ components of the velocity, then $\left.u\right|_{r=r_{0}}=0,\left.r\right|_{r=r_{0}}=0,\left.w\right|_{r=r_{0}}=\omega r_{0} \sin \theta$. System (1) is put in the spherical coordinate system as

$$
\begin{gather*}
u \frac{\partial u}{\partial r}+\frac{v}{r} \frac{\partial u}{\partial \theta}-\frac{v^{2}+w^{2}}{r}+\frac{\partial p}{\partial r}=-\frac{\mathrm{G}^{2}}{\operatorname{Re} \mathrm{Re}_{m}} \frac{1}{r}\left(H_{\varphi}^{2}+H_{\theta}^{2}+r H_{\varphi} \frac{\partial H_{\varphi}}{\partial r}\right. \\
\left.\div r H_{\theta} \frac{\partial I_{\theta}}{\partial r}-H_{\theta} \frac{\partial H_{r}}{\partial \theta}\right)+\frac{1}{\operatorname{Re}}\left[\Delta u-\frac{2}{r^{2}} \frac{\partial v}{\partial \theta}-\frac{2}{r^{2}}\left(u+\frac{v}{\operatorname{tg} \theta}\right)\right], \\
\frac{u v}{r}+\frac{r}{r} \frac{\partial v}{\partial \theta}-\frac{w^{2}}{r \operatorname{tg} \theta}+u \frac{\partial v}{\partial r}+\frac{1}{r} \frac{\partial p}{\partial \theta}=\frac{\mathrm{G}^{2}}{\operatorname{Re} \operatorname{Re}} \frac{1}{r}\left(H_{r} H_{\theta}+r H_{r} \frac{\partial H_{\theta}}{\partial r}\right. \\
\left.-H_{r} \frac{\partial H_{r}}{\partial \theta}-\frac{H_{\varphi}^{2}}{\operatorname{tg} \theta}-H_{\varphi} \frac{\partial H_{\varphi}}{\partial \theta}\right)+\frac{1}{\operatorname{Re}}\left(\Delta v+\frac{2}{r^{2}} \frac{\partial u}{\partial \theta}-\frac{v}{r^{2} \sin ^{2} \theta}\right),  \tag{3}\\
u \frac{\partial u}{\partial r}+\frac{v}{r} \frac{\partial w}{\partial \theta}+\frac{u w}{r}+\frac{v w}{r} \frac{1}{\operatorname{tg} \theta}=\frac{G^{2}}{\operatorname{Re} \operatorname{Re}_{m}} \frac{1}{r}\left(H_{\theta} \frac{\partial H_{\varphi}}{\partial \theta}+\frac{H_{\theta} H_{\varphi}}{\operatorname{tg} \theta}\right. \\
\left.+H_{r} H_{\varphi}+r H_{r} \frac{\partial H_{\varphi}}{\partial r}\right)+\frac{1}{\operatorname{Re}}\left(\Delta w-\frac{w}{r^{2}} \frac{1}{\sin ^{2} \theta}\right), \\
\frac{1}{r} \frac{\partial}{\partial r}\left(r^{2} H_{r}\right)+\frac{1}{\sin \theta} \frac{\partial}{\partial \theta}\left(H_{\theta} \sin \theta\right)=0, \frac{1}{r} \frac{\partial}{\partial r}\left(r^{2} u\right)+\frac{1}{\sin ^{\theta} \theta} \frac{\partial}{\partial \theta}(v \sin \theta)=0, \\
v H_{\varphi}-w H_{\theta}-\frac{1}{r \sin \theta} \frac{1}{\operatorname{Re} e_{m}} \frac{\partial\left(H_{\varphi} \sin \theta\right)}{\partial \theta}=\frac{\partial \Psi}{\partial r}, \\
w H_{r}-u H_{\varphi} \div \frac{1}{r} \frac{1}{\operatorname{Re} e_{m}} \frac{\partial\left(r I_{\varphi}\right)}{\partial r}=\frac{1}{r} \frac{\partial \Psi}{\partial \theta}, u H_{\theta}-v H_{r}-\frac{1}{r} \frac{1}{\operatorname{Re} e_{m}}\left(\frac{\partial r H_{\theta}}{\partial r}-\frac{\partial H_{r}}{\partial \theta}\right)=0,
\end{gather*}
$$

where $G$, Re, and $\operatorname{Re}_{\mathrm{m}}$ are the Hartman and Reynolds numbers, while $\Delta$ is the Laplace operator. All the quantities in (3) may be reduced to dimensionless form by dividing by the characteristic quantities; $r_{0}$, the radius of the sphere; $v=\omega r_{0}$, the rotational speed of an equatorial point on the sphere; and $H_{0}$, the dipole magnetic field at the surface of the sphere in the equatorial plane. The function $\Psi$, which has the meaning of the dimensionless electrostatic field potential, is undefined. Only two of the last three equations are independent.

We divide the entire region of plasma flow into two parts: a boundary region, where the radial gradients in the velocities are large, and a remote region, where the rates of change in the functions are of the same order in all directions.

Near the rotating sphere, we transform the initial system of equations to a form in which the derivatives are of the same order. For this purpose we make a change of independent variable of the form $(r-1) G=\rho$, and instead of the Hartman number $G$, which is subsequently considered as large ( $G \gg 1$ ), we introduce the small parameter $\varepsilon=1 / G$. Then the system of equations for the boundary region may be put as

$$
\begin{aligned}
& \varepsilon^{2}(1+\varepsilon \rho)^{2} u \frac{\partial u}{\partial \rho}+\varepsilon^{3}(1+\varepsilon \rho)\left(v \frac{\partial u}{\partial \theta}-v^{2}-w^{2}\right) \div(1+\varepsilon \rho)^{2} \varepsilon^{2} \frac{\partial \partial}{\partial \rho} \\
= & -\frac{1+\varepsilon \rho}{\operatorname{ReRe} e_{m}} \varepsilon\left(H_{\Phi}^{2}+H_{\theta}^{2}-H_{\theta} \frac{\partial H_{r}}{\partial \theta}\right)-\frac{(1+\varepsilon \rho)^{2}}{\operatorname{ReRe}_{m}}\left(H_{\varphi} \frac{\partial H_{\Phi}}{\partial \rho}+H_{\theta} \frac{\partial H_{\theta}}{\partial \rho}\right)+
\end{aligned}
$$

$$
\begin{aligned}
& +\frac{1}{\operatorname{Re}}\left[2 \varepsilon^{2}(1+\varepsilon \rho) \frac{\partial u}{\partial \rho}+\varepsilon(1+\varepsilon \rho)^{2} \frac{\partial^{2} u}{\partial \rho^{2}}-2 \varepsilon^{3} \frac{\partial v}{\partial \theta}-2 \varepsilon^{3} u\right. \\
& \left.-2 \varepsilon^{3} \frac{v}{\operatorname{tg} \theta}+\varepsilon^{3} \frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial u}{\partial \theta}\right], \\
& \varepsilon^{2}(1+\varepsilon \rho)^{2} u \frac{\partial v}{\partial \rho}+(1+\varepsilon \rho) \varepsilon^{3}\left(u v+v \frac{\partial v}{\partial \theta}-\frac{w^{2}}{\operatorname{tg} \theta}+\frac{\partial p}{\partial \rho}\right)=\frac{(1+\varepsilon \rho)^{2}}{\operatorname{ReRe}_{m}} H_{\mathrm{s}} \frac{\partial H_{\theta}}{\partial \rho} \\
& +\frac{\varepsilon(1+\varepsilon \rho)}{\operatorname{ReRe} \mathrm{Re}_{m}}\left[H_{r} H_{\theta}-H_{r} \frac{\partial H_{r}}{\partial \theta}-\frac{H_{\varphi}^{2}}{\operatorname{tg} \theta}-H_{\varphi} \frac{\partial H_{\varphi}}{\partial \theta}\right]+\frac{\varepsilon(1+\varepsilon \rho)^{2}}{\operatorname{Re}} \frac{\hat{\sigma}^{2} v}{\partial \varphi^{2}} \\
& +\frac{2(1+\varepsilon \rho) \varepsilon^{2}}{\operatorname{Re}} \frac{\partial v}{\partial \rho}+\frac{\varepsilon^{3}}{\operatorname{Re}}\left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial v}{\partial \theta}+2 \frac{\partial u}{\partial \theta}-\frac{v}{\sin ^{2} \theta}\right], \\
& \varepsilon^{2}(1+\varepsilon \rho)^{2} u \frac{\partial w}{\partial \rho}+\varepsilon^{3}(1+\varepsilon \rho)\left(v \frac{\partial w}{\partial \theta}+u w+\frac{v w}{\operatorname{tg} \theta}\right)= \\
& =\frac{(1+\varepsilon \rho)^{2}}{\operatorname{ReRe} e_{m}} H_{r} \frac{\partial H_{\Phi}}{\partial \rho}+\frac{1+\varepsilon \rho}{\operatorname{ReRe}_{m}} \varepsilon\left(H_{\theta} \frac{\partial H_{\varphi}}{\partial \theta}+\frac{H_{\theta} H_{\varphi}}{\operatorname{tg} \theta}+H_{r} H_{\varphi}\right)+\frac{(1+\varepsilon \rho)^{2}}{\operatorname{Re}} \varepsilon \frac{\partial^{2}{ }^{2}}{\partial \rho^{2}} \\
& +\frac{2(1+\varepsilon \rho)}{\operatorname{Re}} \varepsilon^{2} \frac{\partial w}{\partial \rho}+\frac{\varepsilon^{3}}{\operatorname{Re}}\left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \sin \theta \frac{\partial w}{\partial \theta}-\frac{w}{\sin ^{2} \theta}\right], \\
& \frac{\partial}{\partial \rho}(1+\varepsilon \rho)^{2} H_{r}+\varepsilon \frac{1+\varepsilon \rho}{\sin \theta} \frac{\partial}{\partial \theta} H_{\theta} \sin \theta=0, \\
& \frac{\partial}{\partial \rho}(1+\varepsilon \rho)^{2} u+\varepsilon \frac{1+\varepsilon \rho}{\sin \theta} \frac{\partial}{\partial \theta} \nu \sin \theta=0, \\
& \varepsilon(1+\varepsilon \rho)\left(c H_{\varphi}-w H_{\theta}\right)-\frac{\varepsilon}{\operatorname{Re}_{m}} \frac{1}{\sin \theta} \frac{\partial H_{\varphi} \sin \theta}{\partial \theta}=(1+\varepsilon \rho) \frac{\partial \Psi}{\partial \rho}, \\
& \varepsilon(1+\varepsilon \rho)\left(w H_{T}-u H_{\varphi}\right)+\frac{1}{\mathrm{Re}_{m}} \frac{\partial(1+\varepsilon \rho) H_{\varphi}}{\partial \rho}=\varepsilon \frac{\partial \Psi}{\partial v}, \\
& \varepsilon(1+\varepsilon \rho)\left(u H_{\theta}-v H_{r}\right)-\frac{1}{\operatorname{Re}_{m}} \frac{\partial H_{\theta}(1+\varepsilon \rho)}{\partial \rho}+\frac{\varepsilon}{\operatorname{Re}_{m}} \frac{\partial H_{r}}{\partial \theta}=0 .
\end{aligned}
$$

The solution is sought as a series in the parameter $\varepsilon$. We use the zeroth and first approximations of this parameter to get the following expressions for the velocities and components of the magnetic field that satisfy the above boundary conditions:

$$
\begin{aligned}
v & =0, u=0, \\
w & =\frac{1}{\mathrm{G}} \sin \theta \begin{cases}\exp \left(-H_{r}^{0} \rho\right), & H_{r}^{0}>0, \\
\exp \left(H_{r}^{0} \rho\right), & H_{r}^{0}<0,\end{cases} \\
H_{r} & =H_{r}^{0}+\frac{1}{\mathrm{G}}\left\{6 \rho \cos \theta+h_{r}^{1}-2 \rho h_{r}^{0}-\frac{\rho}{\sin \theta} \frac{d}{d \theta}\left(h_{\theta}^{0} \sin \theta\right)\right\}, \\
H_{\theta} & =H_{\theta}^{0}+\frac{1}{\mathrm{G}}\left\{3 \rho \sin \theta+h_{\theta}^{\mathrm{i}}+\rho h_{\theta}^{0}-\rho \frac{d h_{r}^{0}}{d \theta}\right\}, \\
H_{\Phi} & =\frac{\mathrm{Re}_{m}}{\mathrm{C}} \begin{cases}1-\exp \left(-H_{r}^{0} \rho\right), & H_{r}^{0}>0, \\
-1+\exp \left(H_{r}^{0} \rho\right), & H_{r}^{0}<0,\end{cases}
\end{aligned}
$$

where $H_{r}^{0}=-2 \cos \theta+h_{r}^{0}, H_{\theta}^{0}=-\sin \theta+h_{\theta}^{\circ}$, and $h_{\alpha}^{0}$ and $h_{\alpha}^{2}$ are the boundary values of the $\alpha$ component of the perturbed magnetic field in the sphere corresponding to the zeroth and first orders in $\varepsilon$.

From these expressions we draw conclusions on the structure of the flows and electromagnetic fields near the rotating sphere. The speed of the plasma in $r$ or $\theta$ is much less than the speed in $\varphi$. The speed in $\varphi$ due to rotation of the sphere is maximal in the equatorial plane and falls as the poles are approached. The thickness of the Hartman layer, in which the motion is appreciable; is defined by $\delta \sim \frac{r_{0}}{G\left|H_{r}^{0}\right|}$. As $H_{r}^{\circ}$ is dependent on $\theta$, the layer thickness varies with $\theta$. There is an $H_{\varphi}$-component in the magnetic field because of the currents $r$ and $\theta$. These expressions can be simplified for weak perturbations, and $H_{T}^{0} \simeq-2 \cos \theta$, $H_{\theta}^{0} \simeq-\sin \theta$; we merely note that for $H_{r}^{0}$ the approximation does not apply near $\theta \simeq \pi / 2$, while for $H_{\theta}^{0}$ is does not apply near $\theta \simeq 0$. If the perturbations are small, the regions in which the approximation is not applicable are also small.

For the other parameters we have

$$
\begin{aligned}
& \dot{w}=\left\{\begin{array}{l}
\sin \theta \cdot \exp (2 \rho \cos \theta), \frac{\pi}{2}<\theta \leqslant \pi, \\
\sin \theta \cdot \exp (-2 \rho \cos \theta), \quad 0 \leqslant \theta<\frac{\pi}{2},
\end{array}\right. \\
& H_{\mathbf{r}}=-2 \cos \theta+\frac{6 \rho}{\mathrm{G}} \cos \theta_{\mathrm{s}} \\
& H_{\theta}=-\sin \theta+\frac{3 \rho}{\mathrm{G}} \sin \theta, \\
& H_{\varphi}=\frac{\mathrm{Re}_{m}}{\mathrm{G}}\left\{\begin{array}{l}
1-\exp (2 \rho \cos \theta), \quad \frac{\pi}{2}<\theta \leqslant \pi, \\
-1+\exp (-2 \rho \cos \theta), \quad 0 \leqslant \theta<\frac{\pi}{2},
\end{array}\right. \\
& \delta \sim r_{0}(\mathrm{G} 2 \cos \theta)^{-1} .
\end{aligned}
$$

The latter expression implies that the thickness of the Hartman layer is minimal at the poles and increases towards the equator. The thickness at the equator itself is given by $\delta \sim \frac{r_{0}}{\left|h_{r}^{0}\right|} \frac{1}{\mathrm{G}}$.

The magnetic perturbation in the $\varphi$ component changes sign on passing through the equator, which is due to the currents flowing in the meridional plane. For $\theta \in(\pi / 2,0)$

$$
\begin{gathered}
I_{r}=\frac{c \mathrm{Re}_{m}}{4 \pi r \mathrm{G}}\left\{\cos \theta\left(1-\mathrm{e}^{-2 \rho \cos \theta}\right)-\rho \sin ^{2} \theta \cdot \mathrm{e}^{-2 \rho \cos \theta}\right\}, \\
I_{\theta}=-\frac{c \mathrm{Re}_{m}}{4 \pi r \mathrm{G}} \sin \theta\left\{1-\mathrm{e}^{-2 \rho \cos \theta}+2 \rho \cos \theta \cdot \mathrm{e}^{-2 \rho \cos \theta}\right.
\end{gathered}
$$

The current $I_{\theta}$ becomes zero at points $\theta=0$, i.e., at the pole, and tends to zero at the equator. For $\rho \rightarrow 0$ the current is $I_{\theta} \rightarrow 0$ and is directed from the equator to the pole. The radial current becomes zero at the surface of the sphere in accordance with the requirement that no current flows through the surface. For $\theta \rightarrow \pi / 2$ we have $I_{r}<0$, and for $\theta \rightarrow 0$, we have $I_{r}>0$, so the current flows away from the surface of rotation near the pole. Therefore, the radial current becomes zero at an intermediate point $\theta *$. This point is given by

$$
\begin{equation*}
\cos \theta^{*}\left(1-\mathrm{e}^{-2 \rho \cos \theta^{*}}\right)=2 \rho \sin ^{2} \theta^{*} \cdot \mathrm{e}^{-2 \rho \cos \theta^{*}} . \tag{3}
\end{equation*}
$$

Near the surface of the sphere, this equation becomes $\cos ^{2} \theta^{*}=\sin ^{2} \theta^{*}$, so $\theta^{*}=\pi / 4$ and $\theta^{*}=$ $3 \pi / 4$; for $\rho \rightarrow \infty$ we have $\theta^{*} \rightarrow \pi / 2$, although this analysis cannot be used near the equatorial plane. However, the law of conservation for the total current indicates that the current flows into the Hartman layer near the equatorial plane, then flows along the surface, and flows out near the poles. At the outer boundary of the layer $(\rho \rightarrow \infty) I_{r} \simeq c \operatorname{Re}_{m} \cos \theta /\left[4 \pi\left(r_{0}+\delta\right) \mathrm{G}\right]$. Therefore, the current flows out of the Hartman layer along almost all of the external boundary, apart from a narrow band near the equatorial plane, where it flows in. The density of the inflowing current must be much greater than that of the outflowing one. The value can readily be estimated if it is known that $\left|\mathrm{h}_{\mathrm{r}}^{\circ}\right|=\left|\mathrm{H}_{\mathrm{D} \mathbf{r}}^{\circ}\right|$ at $\theta=\theta_{0}$. For the steady-state case we have

$$
\langle I\rangle D \simeq \frac{c \mathrm{Re}_{m}}{4 \pi \mathrm{G}} \int_{\theta_{0}}^{\pi / 2} \frac{\cos \theta d \theta}{\left(r_{0}+\delta\right)}, \text { where } D \simeq r_{0} \theta_{0} .
$$

We put $\delta \ll r_{0}$ for the purpose of estimation, which gives $\langle I\rangle \simeq c \operatorname{Re}_{m} / 4 \pi r_{0} D G$.
This analysis of the equations near a rotating sphere also gives an idea of the plasma behavior at large distances. If the plasma has isotropic scalar conductivity, the currents flowing from the Hartman layer at $\theta>\theta_{0}$ diverge through the plasma the more strongly the larger the Hartman layer. These currents return to the Hartman layer along the equatorial plane, where they flow perpendicular to the lines of force of the magnetic field. The current system and the corresponding electric fields outside the Hartman layer can readily be determined if we neglect the motion of the plasma in this region. Then Ohm's law can be written as $I=\sigma E$. As div $I=0$ and rot $E=0$, we introduce the scalar electric field potential $\Psi: E=-\nabla \Psi$, which gives us the equation $\Delta \Psi=0$ with the boundary condition $\partial \Psi /\left.\partial n\right|_{\Gamma}=I_{n} / \sigma$, where $\Gamma$ is the boundary of the Hartman layer, $n$ is the normal to this boundary, and $I_{n}$ is the current flowing into or out of the layer and which is determined from the previous solution.

This consideration of the currents in the Hartman layer goes with the problem formulated above for the external currents and electric fields to give rise to some important consequences:

1) The density of the outflowing currents varies smoothly along the outer boundary of the Hartman layer;
2) the current flows into the Hartman layer within a narrow band of width $D$ defined by $\left|H_{D r}^{\circ}\right| \sim\left|h_{r}^{\circ}\right|$;
3) the density of the inflowing current is much higher than that of the outflowing one, and as regards order of magnitude we can write $I_{\text {in }} / I_{\text {out }} \approx \mathrm{D} / \mathrm{r}_{0}$;
4) the inflowing current is perpendicular to the lines of force of the dipole magnetic field;
5) the force ( $\mathbf{I} \times \mathbf{H}$ )/sec in the region of the inflowing currents accelerates the plasma in the sense of rotation. The force is directed in the opposite sense in the region of the outflowing currents. Equation (4) defines the line dividing the regions of inflow and outflow;
6) the density of the inflowing currents is much higher than that of the outflowing ones, so the force ( $\mathbf{I} \times \mathbf{H}$ )/sec has the main influence on the motion of the plasma near the equatorial plane;
7) this current structure outside the Hartman layer means that the plasma in the region of the equatorial plane rotates much more rapidly than the plasma outside this region; and
8) the characteristic scale of the region in which the currents are appreciable is of the order of $r_{0}$.

This analysis gives a clear conception of the system of motions, currents, and electric fields near a rotating sphere surrounded by a plasma in the presence of a dipole magnetic field. The results can be used to interpret data on the electrodynamic structure of space near rotating stars and planets having their own magnetic fields.

It is familiar that the speeds of rotation of the surface layers on the sun differ and are dependent on the distance to the equator, where the value is maximal [2]. It is possible that the mechanism is related to currents flowing through the solar plasma and arising from the rotation of the sun and the friction of the upper layers on the corona at rest.

Another example having a bearing on this is the behavior of the plasma near Jupiter [3]. In the equatorial plane, the magnetic field lines are highly elongated as a result of the rapid rotation of the equatorial plasma along with Jupiter.

The electrodynamic structure of inner space is determined by many factors [4, 5], including the earth's rotation. It is difficult to make direct use of the above results to explain experimental data on the motion of the plasma near the earth, but in [6] a current system was derived analogous to that described above on the basis of viscous friction and the earth's rotation.

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